Section-A (MCQ's)

Q.1 Choose the correct answer for each from the given options:

(i) $\int \frac{ax+b}{ax^2+2bx+c}dx =$ _____

(a) $2\ln|ax^2+2bx+c|+c$ (b) $\ln|ax+b|+c$ (c) $\ln|ax^2+2bx+c|+c$

(D) $\frac{1}{2}\ln|ax^2+2bx+c|+c$

(ii) If $A=\int_0^{\pi}\sqrt{1-\cos^2 x}\,dx$, then A = _____.

(a) 0 (b) 2 (c) -2 (d) None of these

(iii) The point of concurrent of attitutdes of a triangle is called _____

(a) Circum-centre (b) In-centre (c) Centroid (d) Orthocentre

(Iv) $ax^2+2hxy+by^2=0$ represent a pair of orthogonal lines if sum of the coefficients of x and y is:

(a) 1 (b) 0 (c) -1 (d) 2

(v) If $90°<\theta<180°$, then slope is:

(a) Undefined (b) Positive (c) Negative (d) Same

(vi) $\lim_{x\to 0}(1-3x)^{\frac{2}{x}} =$ _____

(a) e^6 (b) e^{-6} (c) e^{-3} (d) e^3

(vii) A function f: A $\rightarrow$ B is said to be _____, if the range of f is B.

(a) Bijective (b) Surjective (c) Injective (d) None of these

(Viii) $\int\frac{1}{x\ln x}dx =$ _____.

(a) $\ln|\ln x|+c$ (b) $\ln|\ln x|+c$ (c) $\ln x+c$ (d) $\frac{1}{x}\ln x+c$

(ix) The distance between two parallel lines $4x+3y+2=0$ and $4x+3y+8=0$ is:

(a) $\frac{13}{2}$ (b) $\frac{59}{2}$ (c) $\frac{6}{5}$ (d) $\frac{2}{5}$

(x) If the slope of tangent is $\sqrt{3}$, then the angle is:

(a) 30 (b) 60 (c) 45 (d) None of these

(xi) $(1,5)\cap[3,7]$

(a) $(1,7]$ (b) $(3,5)$ (c) $[3,5)$ (d) $(3,5]$

(Xii) If $f(x)=\sqrt{x+2}$ then range of f is:

(a) $[0,\infty)$ (b) $(-\infty,+\infty)$ (c) $(-2,+\infty)$ (d) $[-2,\infty)$

(xiii) If $y = 5e^{3x-4}$, then $\frac{dy}{dx} =$ _____.

(a) $15e^{3x-4}$ (b) $5e^{3x+4}$ (c) $\frac{e^{3x-4}}{3}$ (d) $(3x-4)e^{3x-4}$

(Xiv) The vertex of the parabola $y = 5(x-2)^2 + 7$ is:

(a) (2, 0) (b) (2, -7) (c) (-2, 7) (d) (2, 7)

(xv) The set $S(c, r) = \{(x, y): |z - cp| = r\}$ contains centre c if:

(a) $r = 0$ (b) $r < 0$ (c) $r > 0$ (d) $r = \infty$

(Xvi) $\int e^{3x}(3\sin x + \cos x)dx =$ _____.

(a) $e^{3x}\sin x + c$ (b) $3e^{3x}\sin x + c$ (c) $e^{3x}\cos x + c$ (d) None fo these

(xvii) If $\vec{u}, \vec{v}, \vec{w}$ are contermious edges of aparallelpiped, then its volume is:

(a) $|\vec{u} \times \vec{v}|$ (b) $[\vec{u}\ \vec{v}\ \vec{w}]$ (c) $\vec{u}.\vec{v}.\vec{w}$ (d) $\vec{u} \times \vec{v} \times \vec{w}$

(xviii) The centre of circle $3x^2 + 3y^2 - 6x + 12y - 7 = 0$ is·

(a) 2, -4) (b) 1, -2) (c) -2, 4) (d) (-1, 2)

Section-B

Note: Solve anyTEN of the following questions. Each question carries 05 marks.

Q.2 Define " Function" and COMPOSITE FUNCTION". Find the domain and range of f, where $f(x) = \sqrt{x^2 + 9}$

Q.3 Evaluate: $\lim_{x \to 0} \frac{x(\cot x - \operatorname{cosec} x)}{\sin x}$

Q.4 Find the equation of a line through the intersection of lines $3x - 4y + 1 = 0$, $5x + y - 1 = 0$ and cutting off equal intercepts from the axes.

Q.5 Show that the equation of the line through the origin making an angle of measure ϕ with the line $y = mx - b$ is $\frac{y}{x} = \frac{m - \tan\phi}{1 + m\tan\phi}$

Q.6 find $\frac{dy}{dx}$ when, $y = \sqrt{\ln(e^{2x} - e^{-2x})}$

Q.7 Relative to an origin 0, the position vectors of the points A and B are given by $2\vec{i} + 3\vec{j} + \vec{k}$ And $4\vec{i} - 3\vec{j} + 2\vec{k}$ respectively. Use scalar product to find angle AOB correct to nearest degree.

Q.8 Find the relative maximum and minimum values of the function $f: \mathbb{R} \to \mathbb{R}$ $\inf(x) = -x^4 + 2x^2$

.9 Solve: $2 + 2y\frac{dy}{dx} = 1 + 3x^2, y(2) = 1$

Q.10 Find an equaiton fo the circle pasisng through the points A (1, 2) and B(1, -2) and touching the line $x + 2y + 5 = 0$.

Q.11 Find the coordinates of the centre and the foci, length of the semi-transverse axis, the eccentricity, and the equations of the directices of the hyperbola $9x^2 - 4y^2 + 36x + 8y - 4 = 0$

Q.12 Find $\frac{dy}{dx}$ if, $x^{3y} \cdot y^{3x} = 9$.

Q.13 Evaluate: $\int e^x\left(\frac{1 + \sin x}{1 + \cos x}\right)dx$

Q.14 A sequence in given by $\frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \ldots$ write down the general term of the sequence and find its limit.

Section-C

Note: Solve any THREE of the following questions.

Q.15 (a) Show that: $\int_3^4 \frac{3x}{(x+1)(x-2)}dx = \ln 5$.

(b) Find the gradient of the curve $y = \frac{12}{x^2 - 4x}$ at the point where $x = 3$.

Q.16 (a) Find the condition that the conic $ax^2 + by^2 = 1$ should cut $a'x^2 + b'y^2 = 1$ Orthogonally.

(b) A curve has equation $y = \frac{4}{\sqrt{x}}$. Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

Q.17 (a) Find the equation of the ellipse, whose centre is origin and directrix = 16, length of latus rectum 12.

(b) The equation of a curve is ln (xy) -y3 =1. Show that $\frac{dy}{dx} = \frac{y}{x(3y^3 - 1)}$

Q.18 (a) If $y = \frac{1}{x^2}$, then find $\frac{dy}{dx}$ at $x = -1$ by ab-initio method.

(b) Find the area of the triangle with vertices A(1,-1,1), B(2, 1, -1) and C(-1,1,2). Also find a unit vector perpendicular to the plane ABC.