

Section-A

Multiple Choice Questions (MCQ's)

Q.1: Choose the correct answer for each form the given options:

- (i) The function defined by $f(x) = |x|$ is called:
 (a) Rational function (b) Constant function (c) Circular function
 (d) Modulus function
- (ii) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = -x$ is:
 (a) An even function (b) An odd function (c) Neither even nor odd
 (d) None of these
- (iii) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n =$ _____
 (a) e (b) 0 (c) ∞ (d) 1
- (iv) $\lim_{x \rightarrow 0} \frac{3^x - 1}{x} =$ _____
 (a) e (b) $\ln 3$ (c) $\log_3 3$ (d) ∞
- (v) The point of concurrence of the internal bisectors of the angles of a triangle is called
 its:

- (a) Centroid (b) Circum centre (c) In centre (d) Ortho centre

- (i) The inclination of a line parallel to x - axis is:
 (a) 0° (b) 90° (c) 45° (d) 30°
- (ii) The distance between (-2, -1) and (9, 4) is:
 (a) $\frac{1}{13}$ (b) 11 (c) 13 (d) 15
- (iii) The angle between the lines represented by $3x + 7y + 2z = 0$ is:
 (a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{6}}{6}$ (c) $\frac{\sqrt{9}}{9}$ (d) $\frac{\sqrt{4}}{4}$
- (x) If $y = a$, then $\frac{dy}{dx} =$ _____
 (a) $\frac{1}{x}$ (b) $a \ln a$ (c) $\frac{1}{a \ln x}$ (d) $\frac{1}{x \ln a}$
- (y) $\frac{d}{dx} \sin^{-1} \frac{a}{x} =$ _____
 (a) $\frac{-1}{x\sqrt{x^2 - a^2}}$ (b) $\frac{-a}{x\sqrt{x^2 - a^2}}$ (c) $\frac{a}{x\sqrt{x^2 - a^2}}$ (d) $\frac{1}{\sqrt{x^2 - a^2}}$
- (z) $\int \frac{du}{u^2} =$ _____
 (a) $u + c$ (b) $-\frac{1}{u}$ (c) $-\frac{1}{u} + c$ (d) 0
- (AA) $\int \frac{(\ln x)^2}{x} dx =$ _____
 (a) $\frac{(\ln x)^4}{4} + c$ (b) $\ln x + c$ (c) $(\ln x) + c$ (d) $\frac{1}{x^3} + c$
- (AB) The centre of the circle $(x + 2)^2 + y^2 = 25$ is:
 (a) (0, 0) (b) (0, -2) (c) (-2, 0) (d) (-2, 5)
- (AC) Length of the semi latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:
 (a) $2b/2$ (b) b/a (c) $2b^2/a$ (d) b^2/a^2
- (AD) The eccentricity of a rectangular Hyperbola is:
 (a) 1 (b) 2 (c) $\frac{2}{\sqrt{2}}$ (d) $\frac{1}{2}$

- (xvi) The radius of the circle $x^2 + y^2 + 2fy + c = 0$ is:
 (a) $\sqrt{g^2 + f^2 - c}$ (b) $\frac{1}{2}\sqrt{g^2 + f^2 - c}$ (c) $\frac{1}{2}\sqrt{g^2 + f^2 + c}$ (d) $\sqrt{g^2 + f^2 + c}$
- (xvii) The unit vector of a vector $(3m - 2, 7)$ is:
 (a) $\frac{2i + 3j + 7k}{\sqrt{62}}$ (b) $\frac{3i + 2j + 7k}{\sqrt{54}}$ (c) $\frac{3i + 2j + 7k}{\sqrt{64}}$ (d) $\frac{3i + 2j + 7k}{\sqrt{62}}$
- (xviii) If $a = 3i - xj + 2k$ and $b = 4i + 3k$ are perpendicular then $x =$ _____
 (a) 18/7 (b) 2/3 (c) 3 (d) 6
- (xix) The focal radius of the point (a, b) on the parabola $x = 4xy$ equal to:
 (a) $|a - b|$ (b) $|a + b|$ (c) $|b - c|$ (d) $\frac{1}{2}|a + b|$
- (Xx) $\int_0^{x/3} \sin 2x \, dx =$ _____
 (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 1 (d)

Section-B

Note: Solve any TEN of the following questions. Each question carries 5 marks

- Q.2 A function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = \begin{cases} 0 & \text{when } x \in \mathbb{Q} \\ 1 & \text{when } x \in \mathbb{R} - \mathbb{Q} \end{cases}$ then prove that the function is neither one-one nor onto. Also find the range of f .
- Q.3 The points $(1, 2)$ and $(3, 4)$ are the mid points of the sides of a triangle. Find its vertices.
- Q.4 Evaluate $\lim_{x \rightarrow 0} \left(\frac{1 - 2x}{1 - 3x} \right)^{\frac{1}{x}}$
- Q.5 Prove that diagonals of an isosceles trapezoid are equal.
- Q.6 Use differentials, approximate $\tan 46^\circ$.
- Q.7 Find the fourth vertex of the parallelogram formed by the points $(1, -2)$, $(1, 0)$ and $(2, 1)$.
- Q.8 Show that an equilateral triangle has congruent angles.
- Q.9 Find the tangents of the angles of the triangle whose vertices are $A(4, 1)$, $B(-1, 3)$ and $C(-5, 2)$.
- Q.10 Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 4}}{x + 2}$
- Q.11 Solve the differential equation $4 + 3y^2 \frac{dy}{dx} = 1 + 2x^2$, when $y(3) = 1$.
- Q.12 Find the equation of the circle touching both the axes and of radius 5 units in the first and fourth quadrants.

- Q.13 If $\int_0^{\pi/2} k \cos x \, dx = 4$, find k ?
- Q.14 For what value of p , do the vectors $p\hat{i} + 2\hat{j} + k\hat{k}$, and $-2\hat{i} + \hat{j}$ are coplanar.
- Section-C**
- Note: Solve any THREE of the following questions.**
- Q.15 (a) If $y = (\sin x)^3$ prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$
 (b) Find the volume of the parallelepiped determined by $\vec{u} = 3\hat{i} + 2\hat{j} + k\hat{k}$, $\vec{v} = \hat{i} - 2\hat{j} + 3k\hat{k}$, $\vec{w} = \hat{i} + 7\hat{j} + 4k\hat{k}$.
- Q.16 Find the derivative of $\tan 2\sqrt{x}$ by definition. (b) Find $\int \frac{e^x(1+x)}{(2+x)^2} dx$.
- Q.17 (a) A line whose y -intercept is 1 less than its x -intercept forms with the coordinate axes a triangle of area 6 square units. What is its equation?
 (b) Use differentials to find approximate value of $\cot 59^\circ$.
- Q.18 (a) Prove that the two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy + c = 0$ touch each other if $\frac{1}{g^2} + \frac{1}{f^2} = \frac{1}{c}$
 (b) Find the directrices and length of latusrectum of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$