Section-A (MCQ's)

Q.1: Choose the correct answer for each form the given options:

(i) If f(x) -2x, g(x) = 1/2 x and fog (x) = 6 then x is:
(a) 4 (b) 12 (c) 6 (d) 2

(ii) $\lim_{x\to 0} \frac{\cos x - 1}{x} =$ ______
(a) 1 (b) 0 (c) -1 (d) 2

(iii) If $f(x) = \frac{3x-4}{2}$, then the valoe of $f^{-1}(x)$ is:
(a) $\frac{3x+2}{4}$ (b) $\frac{2y+3}{4}$ (c) $\frac{2x+4}{3}$ (d) None of these

(iv) If $y = a^x$ (where $a \in \Re^+$ and if $0 < a < 1$ then a^x is ______

(a) Decreasing function
(b) Increasing function
(c) Exponenetial function
(d) Constant function

(v) The centraid G of a ftrinagle divides each median in the ratio ______
(a) 1:2 (b) 2:1 (c) -2:1 (d) 1:1

(vi) If $m_1 = \frac{1}{2}$ and $m_2 = -2$ then $\tan\theta$ will be:
(a) ∞ (b) 10 (c) 45 (d) 90

(vii) The distance of the point (6, -2) from the line 3x -4y + 4 = 0 is ______
(a) -5 units (b) 5 units (c) -6 units (d) -6 units

(viii) $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} =$ ______
(a) f(x') (b) f(h') (c) f'(x) (d) f'(x+h)

(ix) If the slope of the line joining the points (6, 3) and (4, k) is -3, than k is ______
(a) 0 (b) 9 (c) -9 (d) 3

(x) If measure of the angle between the two lines $ax^2 - 2hxy + hy^2 = 0$ is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ then the two line are ______
(a) Parallel (b) Coineident (c) Perpendicular (d) Neither

(xi) $\frac{d}{dx} \sin^{-1} u =$ ______
(a) $\frac{1}{\sqrt{1-u^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-n^2}}$ (d) $\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

(xii) If $y = \sqrt{\sin 5x}$, then $\frac{dy}{dx} =$ ______
(a) $\frac{5\cos 5x}{5}$ (b) $\frac{5\cos 5x}{2\sqrt{\sin 5c}}$ (c) $\frac{1}{2\sqrt{\sin 5x}}$ (d) $\frac{-\sin 5x}{2\sqrt{\cos 5x}}$

(xiii) y = f(x) is increasing if:
(a) $\frac{dy}{dx} < 0$ (b) $\frac{dy}{dx} = 0$ (c) $\frac{dy}{dx} > 0$ (d) $\frac{dy}{dx} = \infty$

(xiv) If a = 3i _ xj + 2k and b = 4i + 3K are perpendicular then x = ______
(a) 18/7 (b) 2/3 (c) 3 (d) 6

(xv) $x^2 + y^2 - 2gx - 2fy + f = 0$ rerp[esent a circle towching the ______
(a) x-axis (b) y-axis (c) origin (d) botht he axis

(xvi) $y^2 = 4ax$ represents a curve sysmeetrical about the ______
(a) x-axis (b) origin (c) y-axis (d) both the axis

Section-B

Note: Solve any TEN of the following questions. Each question carries 05 marks.

Q.2: Show that the sequence $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \ldots\ldots$ is divergent.

Q.3: Show that $\lim_{x\to +\infty} \frac{\ln(1+a^x)}{x} = 1$

Q.4: Show that the point $(\frac{13}{4}, \frac{9}{4})$ is the centre of the circle circumscruibing the triangle whose vertices are (2,1), (5,2) and (3, 4) respectively.

Q.5: Find the combined equation of the pair of lines through the origin which are perpendicular to the lines $2x - 5xy + y = 0$.

Q.6: If $x = y$ find $\frac{dy}{dx}$

Q.7: Calculate log10(10.10) given that $\log_{10} e = 0.4343$.

Q.8: The curve $y = x - 3x - 9x + k$, where k is constant. The curve has a minimum point of the x-axis. Find the value of k, Also find the coordinates of the maximum point of the curve.

Q.9: Use the substitution $x = \sin^2\theta$. Find the exact value of, $\int_0^{3/4} \sqrt{\frac{x}{1-x}}\, dx$

Q.10: Prove analytically a normal to a circle passes through the centre of the circle.

Q.11: Find the equation of the ellipese having the origin as centre, are focus at the point (3,0) and the corresponding directrix $x = 8$.

Q.12: Show that hte eccentricities e_1 and e_2 of the conjugate hyperblas satisfy the relation.
$e_1^2 + e_2^2 = e_1^2 e_2^2$

Q.13: Let $f(x) = \frac{7x+4}{(2x+1)(x+1)^2}$ express f(x) in partial fractions.

Q.14: The vertices of a triangle are the points A:(1,2 1) (-3, 1,1), C:(1,-1,2). Forces of magnitude 3 and 5 act at a along $\vec{AB}$ and $\vec{AC}$ Find their resultant.

Section-C

Note: Attempt any THREE of the following questions.

Q.15(a): Find the derivative of $f(x) = \sec 2x$ by first principle.

(b) Use integration by parts to show that $\int_2^4 \ln x\, dx = 6 \ln 2 - 2$.

Q.16(a): A is the mide piont of the segment bounded by (-4,4) and (2, 2) B is a point at $\frac{3}{5}$ of the distance from (5,3) to (-3, 2). Find the equation of $\overline{AB}$.

Q.17(a) Derive the eqaution fo strlaght line in the normal form x.

(b): Evaluate $\int \cot x \cdot \text{cosec}\, x\, dx$.

Q.18(a): If $y = f(x) = a\,ex + b\,ex + ce$ - for all x show that $\frac{d^2y}{dx^2} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

(b) A cruve is such that $\frac{dy}{dx} = 2x^2 - 5$. Given that point (3, 8) lies on the curve, Find the equatino of the curve.