Sig. of Supdt. ....

KT-XII-1601

MATHEMATICS
(Part – II)
(Fresh / New Course)

Roll No. ....

Fic. #..........................

Fic. # ....

Total Marks: 100

MATHEMATICS

(Part – II)
(Fresh / New Course)

Time Allowed : 3 Hrs.

Marks: 20

Section "A"

Time : 20 Mins.

NOTE : Section-A is compulsory. All parts of this section to be answered on the questions paper itself. It should be completed in the given time and handed over to the Centre Superintendent. Deleting / Overwriting is not allowed. Do not use lead pencil.

NOTE : Insert the correct option (a, b, c, d) in the empty box opposite to each part.

Q. 1 **Insert the correct option (a, b, c, d) in the empty box opposite to each part. Each part carries one mark.**

i) The function $f(x) = x^2$ is ;
(a) Many-to-one (b) One-to-one (c) Onto (d) Both 1-1 and onto

ii) Let $f(x) = 2x + 3$, then inverse of $f(x)$ is ;
(a) $2x - 3$ (b) $\frac{x-3}{2}$ (c) $\frac{2x}{3}$ (d) $3 - 2x$

iii) The average rate of change in y per unit change in x is ;
(a) The slope of the secant line (b) The slope of the tangent line
(c) The derivative of y w.r.t. x (d) The exact rate of change

iv) $\frac{d}{dx}5^{(x+1)} =$ ....
(a) $5^{(x+1)}$ (b) $(x+1)5^x$ (c) 5^x (d) $5^{(x+1)}\ln 5$

v) If $f(x) = \frac{1}{4x+3}$ then $f^{(5)}x =$ ....
(a) $\frac{-4}{(4x+3)^5}$ (b) $\frac{-4^5}{4x+3}$ (c) $\frac{4}{(4x+3)^6}$ (d) $\frac{(-1)^5.5!4^5}{(4x+3)^5}$

vi) $e^1 =$ ....
(a) $1+1+\frac{1}{2!}+\frac{1}{3!}$ (b) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$..... (c) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$..... (d) None of these

vii) Let $f'(c) =$ and $f''(c) => 0$ then the graph $f(x)$ is
(a) Concave up (b) Concave down (c) Straight (d) No information

viii) The domain of $\vec{F}(t) = 2t\,\vec{i} - 3t\,\vec{j} + t^{-1}\vec{k}$ is
(a) $\mathbb{R}$ (b) $2\mathbb{R}$ (c) Q (d) $\mathbb{R} - \{0\}$

ix) $\int 6e^{6t}dt =$ ....
(a) $e^{6t}+c$ (b) $6e^{6t}+c$ (c) $\frac{e^{6t}}{6}+c$ (d) $6+e^{6t}+c$

x) $\int e^x \sin e^x dx =$ ....
(a) $\sin e^x + c$ (b) $-\cos e^x + c$ (c) $e^x + c$ (d) $e^x + \sin e^x + c$

xi) Let f(x) is an odd function and integrable over [–a, a] then $\int_{-a}^{a} f(x)dx =$ ....
(a) 0 (b) $2\int_0^a f(x)dx$ (c) $\int_0^a f(x)dx$ (d) f(x)

xii) The number that represents the "steepness" of a line is called ....
(a) Direction of the line (b) Slope of the line
(c) Dispersion point (d) Intercept

xiii) Two lines are parallel, if the cross products in between the unit vectors U and V is ....
(a) 1 (b) –1 (c) UV (d) 0

xiv) If the angle between two straight line represented by $ax^2 + 2hxy + by^2 = 0$ is 90° then ....

(a) $a + b = 0$ (b) $a = b$ (c) $h^2 = ab$ (d) $h^2 - ab \neq 0$

xv) In $x^2 + y^2 + 2gx + 2fy + c = 0$ of $g^2 + f^2 - c < 0$ then the circle is ....

(a) Point circle (b) Real circle (c) Concentric circle (d) Virtual

xvi) The vertex V (0,0) of the parabola is on the principle axis of symmetry midway between ....

(a) The focus and the directrix (b) The focus and tangent

(c) The directrix and tangent (d) None of these

xvii) The line $y = mx + c$ will be tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if C = ...

(a) $\pm\sqrt{a^2m^2 + b^2}$ (b) $\pm\sqrt{a^2m^2 - b^2}$ (c) $\pm\sqrt{b^2 - a^2m^2}$ (d) $a^2m^2 = b^2$

xviii) For the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the asymptotes are the lines ....

(a) $y = \pm\frac{a}{b}x$ (b) $y = \pm\frac{b}{a}x$ (c) $y = \pm bx$ (d) $y = \pm ax$

x) Degree of the D.E. $\left(\frac{d^3y}{dx^3}\right)^2 - \frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + y = 3$ is ....

(a) 0 (b) 1 (c) 2 (d) 3

xx) For finding area, if approximation is made through parabolic are, then it is known as ....

(a) Rectangular rule (b) Trapezoidal rule

(c) Simpson's rule (d) Newton's rule

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MATHEMATICS

(Part – II)

Total Marks : 80

Time Allowed : 2:40 Hrs.

Section – B

Marks : 50

Q. 2 Write short answers of any TEN of the following parts. Each part carries equal marks.

(i) If $f(x) = 2x + 3$, $g(x) = 3x$ and $h(x) = f(g(x))$, then find $h^{-1}(x)$.

(ii) Evaluate $\lim\limits_{x \to 0} \dfrac{\sec x - 1}{x \sec x}$.

(iii) Differentiate $f(x) = x^2 + 1$ by first principle rule.

(iv) Find the derivative of $\tan^{-1}\sqrt{x}$.

(v) Find the Maclaurin series of $f(x) = \sin x$.

(vi) Evaluate $\int \dfrac{\cos x \,\ell n(\sin x)}{\sin x} dx$.

(vii) Evaluate the integral by partial fraction decomposition $\int \dfrac{x^2 - 1}{x^2 - 2x - 15} dx$

(viii) Find the angle θ from the line L_1 to line L_2, if the slopes of L_1 and L_2 are $m_1 = \dfrac{1}{2}$, $m_2 = 3$.

(ix) Determine the equation of a circle using the given information $C(0,0)$, tangent to the line $x = -5$.

(x) Write the equation of parabola of its focus is $F(0,3)$ directrix $y = -3$.

(xi) Find the general solution of the D.E. $e^x \dfrac{dy}{dx} + y^2 = 0$.

(xii) If $u = f\left(\dfrac{y}{x}\right)$, then show that $x\dfrac{\partial y}{\partial x} + y\dfrac{\partial y}{\partial y} = 0$.

(xiii) Use Simpson's rule to approximate the definite integral and then compare with the exact value of the definite integral $I = \int_2^4 x^2 dx$, $n = 3$.

Section – C

Marks : 30

NOTE : Attempt any THREE questions. Each question carries equal marks.

Q. 3 a) Prove that $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = \ell n a,\ a > 0$

b) Find the 5^{th} derivative of $f(x) = (6x+4)^8$

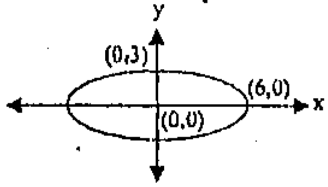
Q. 4 a) Find the angle of intersection between $x^2 - y^2 = a^2$, $x^2 + y^2 = a^2\sqrt{2}$.

b) If ABC is a triangle with vertices $A(0,0)$, $B(8,6)$ and $C(12,0)$, then show that the right bisectors of triangle ABC are concurrent.

Q. 5 a) Use integration by parts to evaluate the integral $\int x \sin x \cos x dx$.

b) Find the equations of the tangents to the circle $x^2 + y^2 = 25$ which are parallel to the straight line $3x + 4y + 3 = 0$

Q. 6 a) Determine the equation of the graphed ellipse.



b) If $U = \tan^{-1} \dfrac{x^2 + y^2}{x + y}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial x} = \tan U$.