

Code = A

Time allowed: 3 Hrs

Mathematics (Part – II)

Marks: 100

Fresh / Reappear

Note: There are three sections of the paper, A, B & C. Attempt Section – A on the same paper and return it to the Superintendent within the given time. No marks will be awarded for cutting, erasing or over writing. Mobile phone etc. are not allowed in the examination hall.

Time: 20 Mins

Section "A"

Marks: 20

Q.1 Write the correct option i.e. A, B, C or D in the empty box provided opposite each part.

- i. For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ the equation $y = \frac{b}{a}x$ is of line ☐ C
- A. Tangent B. Normal C. Asymptote D. Focal chord
- ii. The differential equation $\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + y = 3$ is ☐ C
- A. Homogenous B. Non-linear C. Linear D. Exact
- iii. If $Z = f(x, y)$ is continuously differential homogenous function of degree n then $nz =$ ☐ A
- A. $xf_x + yf_y$ B. $xf_y + yf_x$ C. $xf_y - yf_x$ D. $xf_x - yf_y$
- iv. The slopes of the tangent lines to the graph of the function $f(x)$ to approximate roots of $f(x)$ are used ☐ C
- A. Secant method B. Regula – Falsi method C. Newton – Raphson method D. Simpson method
- v. $\int \frac{dx}{\sqrt{x^2 + a^2}} =$ ☐ A
- A. $\ln|x + \sqrt{x^2 + a^2}|$ B. $\ln|x - \sqrt{x^2 + a^2}|$ C. $\ln|x + \sqrt{x^2 - a^2}|$ D. $\ln|x + \sqrt{a^2 - x^2}|$
- vi. $\int_0^a [f(x) + f(-x)]dx = 2 \int_0^a f(x)dx$ if $f(x)$ is ☐ B
- A. Odd B. Even C. Liner D. Constant
- vii. Centroid is the point through which there pass all the ☐ C
- A. Altitudes B. Angel bisector C. Medians D. Right bisectors
- viii. Concurrent lines which are perpendicular on the sides and passing through vertices are ☐ B
- A. Medians B. Altitudes C. Right bisectors D. Angle bisectors
- ix. $y = x^3$ is ☐ C
- A. one-one-function B. One-to-many function C. Many-to-one function D. Many-to-many function
- x. If the range of a function is singleton set then it is ☐ B
- A. Linear function B. Polynomial function C. Constant function D. Exponential function
- xi. If $y = f(x)$ then $\{f(x)\}$ is the ☐ B
- A. Slope of normal line B. Slope of tangent line C. Slope secant line D. Slope of any chord
- xii. $\frac{d}{dx}(\sec^{-1}x) =$ ☐ A
- A. $\frac{1}{x\sqrt{x^2-1}}$ B. $\frac{1}{x\sqrt{x^2+1}}$ C. $\frac{1}{x\sqrt{1-x^2}}$ D. $\frac{-1}{x\sqrt{x^2-1}}$
- xiii. $x + \frac{x^2}{2!} + 2\frac{x^3}{3!} + \dots$ is the Maclaurin's series expansion of ☐ B
- A. $\ln x$ B. $\ln(1+x)$ C. $\ln(1-x)$ D. e^x
- xiv. If $f(x)$ is strictly decreasing for $x \in (a, b)$ then $f'(x)$ ☐ B
- A. Equal to zero B. Less than zero C. Less than or equal to zero D. has no role
- xv. If $f'(c) = 0$ at $(c, f(c))$ then $f(x)$ has a ☐ D
- A. Critical point B. Maximum point C. Minimum point D. Inflection point
- xvi. If $F(t)$ and $G(t)$ two vector functions then which one is not vector function? ☐ C
- A. $F - G$ B. $F + G$ C. $F \cdot G$ D. $F \times G$
- xvii. The equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $A = B = C = 0$ represents ☐ D
- A. Parabolas B. Circles C. Ellipse D. Straight lines
- xviii. $y = mx + c$ is tangent to $x^2 + y^2 = a^2$ is ☐ A
- A. $c^2 = a^2(1+m^2)$ B. $a^2 = c^2(1+m^2)$ C. $m^2 = a^2(1+c^2)$ D. $c^2 = a^2(m^2-1)$
- xix. Chords of circle which are equidistant from its center are ☐ C
- A. Same B. Equal C. Similar D. Congruent
- xx. If $y^2 = 4px$ is parabola then the equation $x = p$ represents its ☐ B
- A. Directrix B. Latus-rectum C. Axis D. Tangent

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Section "B"**Marks: 50**

Q.2 Attempt any TEN parts. All carry equal marks.

- i. Evaluate $\lim_{x \rightarrow 1} \left(\frac{\frac{1}{x} - 1}{x - 1} \right)$
- ii. Use 1st principle to find the derivative of $f(x) = 3x^2 + 4x - 9$.
- iii. Find $\frac{dy}{dx}$ such that $y = \sec^2(x^2 + 1) + \tanh(x^2 + 1)$
- iv. Find $\frac{d^2y}{dx^2}$ using parametric equations $x = a \frac{1-t^2}{1+t^2}$, $y = b \frac{2t}{1+t^2}$
- v. Let $F(t) = t + tj + t^2k$ and $G(t) = ti + e^j + 3k$ then verify that $\frac{d}{dt}(F \times G) = \frac{dF}{dt} \times G + F \times \frac{dG}{dt}$
- vi. Evaluate $\int e^x \cos x \, dx$.
- vii. Determine the approximate area of the region bounded by $f(x) = 2x + 1$ taking $n = 8$, $a = 0$, $b = 2$
- viii. Find the equation of the straight line through the point of intersection of $2x + y + 1 = 0$, $2x + 3y + 5 = 0$ that touches the point of intersection of $x - y = 0$ and $x + y = 0$.
- ix. Find the equation of circle containing the points $(0, 3)$, $(4, 1)$ and has its center on the x-axis.
- x. Find 'c' so that $x + y + c = 0$ is tangent to $x^2 = -8y$.
- xi. Solve $2 \frac{dy}{dx} = 4xe^{-x}$ s.t $y = 42$ when $x = 0$.
- xii. Let $f(x, y, z) = x^2 ye^{2x} + (x + y - z)^2$ then find $\frac{\partial}{\partial y}(1, y, 1)$ and $\frac{\partial}{\partial z} f(1, 1, z^2)$
- xiii. Find $\sqrt{5}$ by Newton – Raphson iterative method with $x_0 = 2$ correct to 3 decimal places.

Section "C"**Marks: 30**

Note: Answer any THREE questions. Each question carries equal marks.

Q.3.a. Find the critical values of $f(x) = \frac{x^3}{3} - x^2 - 15x + 6$ b. Evaluate $\int \frac{dx}{\sqrt{e^{2x} - 4}}$ Q.4.a. Find the joint equation of the straight line passes through the origin and perpendicular to the lines represented by $x^2 - 2 \tan \theta xy - y^2 = 0$ b. Find the locus of the point P, if the length of the tangent from the point P to the circle $x^2 + y^2 = 9$ is equal to the perpendicular distance from P to the line $3x + 4y + 3 = 0$.Q.5.a. Identify the vertices, foci and asymptotes of $(x - 3)^2 - (y + 1)^2 = 4$.b. Find the solution curve of the differential equation $xy y' = 3y^2 + x^2$ passes through $(-1, 2)$ Q.6.a. Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z$ where $z = xyf(x/y)$ b. Find $\frac{dy}{dx}$ where $y = \frac{1 + \tan 2x}{\operatorname{cosec} 3x}$